

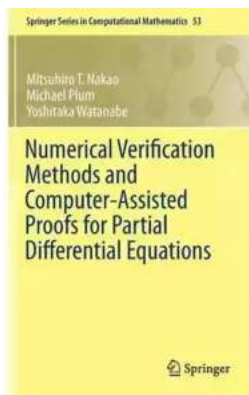
The Fascinating World of Numerical Verification Methods And Computer Assisted Proofs For Partial

Have you ever wondered how mathematicians ensure the accuracy of their calculations and proofs? In the realm of partial differential equations (PDEs), numerical verification methods and computer assisted proofs play a crucial role. These sophisticated techniques enable researchers to validate the solutions obtained through computational simulations and provide reliable mathematical proofs. In this article, we will explore the ins and outs of numerical verification methods and computer assisted proofs for partial differential equations.

Understanding Partial Differential Equations

Before delving into numerical verification methods and computer assisted proofs, let's briefly understand what partial differential equations are. PDEs are mathematical equations that involve multiple variables and their partial derivatives. They describe various physical phenomena, such as heat transfer, fluid dynamics, and electromagnetic fields. Solving PDEs is essential in many scientific and engineering disciplines, enabling researchers to model and analyze complex systems.

However, solving PDEs can be a challenging task due to the complex nature of these equations. Exact analytical solutions are often impossible to obtain, especially for real-world problems. This is where numerical methods come into play, allowing researchers to approximate solutions and validate them through numerical verification methods.



Numerical Verification Methods and Computer-Assisted Proofs for Partial Differential Equations (Springer Series in Computational Mathematics

Book 53) by Yanhua Shih(1st ed. 2019 Edition, Kindle Edition)

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Numerical Verification Methods

Numerical verification methods provide a way to validate the accuracy of computational solutions for PDEs. These methods involve conducting various tests and computations to ensure the correctness of the obtained numerical results. Let's explore some commonly used numerical verification methods:

Method of Manufactured Solutions

The method of manufactured solutions is a powerful technique often used for validating computational methods. It involves constructing a known solution to a PDE and using it to generate artificial data. By applying a numerical algorithm to this artificial data, researchers can compare the computed solution against the known solution. If they match, it provides confidence in the accuracy of the computational method.

For example, let's say we have a PDE representing heat conduction in a particular material. We can generate a known solution by prescribing the exact temperature distribution over the domain. By comparing the computed

temperature distribution using a numerical method with the prescribed solution, we can verify the accuracy of the method.

Convergence Analysis

Convergence analysis is another crucial numerical verification method. It involves studying the behavior of the numerical solution as the computational grid is refined. By decreasing the grid size, the approximation becomes more accurate, and the computed solution should converge to the true solution of the PDE.

Convergence analysis typically involves calculating error indicators, such as the difference between the numerical and exact solutions at various grid resolutions. If the error decreases as the grid size decreases, it provides evidence that the numerical method is converging to the true solution. However, if the error stagnates or even increases, it indicates potential issues with the computational method.

Through convergence analysis, researchers can gain insights into the behavior and accuracy of numerical methods, ensuring the reliability of the obtained solutions.

Computer Assisted Proofs

While numerical verification methods provide a way to validate computational solutions, computer assisted proofs take the verification process to a whole new level. These proofs involve using computer programs to aid in mathematical investigations and ensure the rigor and correctness of mathematical arguments.

For partial differential equations, computer assisted proofs are employed to validate mathematical properties and propositions related to the solutions. Instead of relying solely on analytical or approximate solutions, computer

programs are used to perform rigorous calculations and establish the validity of certain statements.

Computer assisted proofs utilize a range of mathematical techniques, such as interval arithmetic, symbolic computation, and interval analysis. These methods allow researchers to handle complex mathematical expressions and inequalities, ensuring the precision of the obtained results.

By combining numerical verification methods with computer assisted proofs, researchers can greatly enhance the confidence in their computational solutions and mathematical arguments.

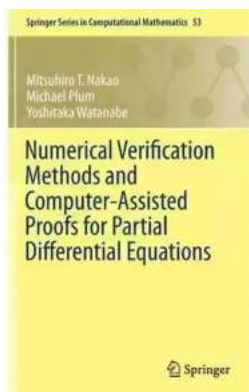
The Importance of Numerical Verification Methods and Computer Assisted Proofs

Numerical verification methods and computer assisted proofs are crucial in various scientific and engineering fields. They provide a means to ensure the accuracy and reliability of computational simulations, enabling researchers to trust the solutions obtained for complex problems.

Moreover, these techniques also contribute to the advancement of mathematics itself. Computer assisted proofs allow researchers to explore new mathematical territories and validate conjectures that were previously unattainable. They provide a tool for investigating intricate mathematical phenomena and deliver robust evidence of the validity of certain mathematical statements.

Numerical verification methods and computer assisted proofs play a pivotal role in the world of partial differential equations. These techniques enable researchers to go beyond mere computational simulations and establish the accuracy and correctness of their results. By combining rigorous numerical verification methods

and computer assisted proofs, mathematicians can confidently explore complex problems, contributing to both applied sciences and mathematics itself.



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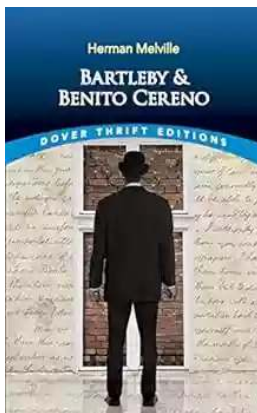


In the last decades, various mathematical problems have been solved by computer-assisted proofs, among them the Kepler conjecture, the existence of chaos, the existence of the Lorenz attractor, the famous four-color problem, and more. In many cases, computer-assisted proofs have the remarkable advantage (compared with a “theoretical” proof) of additionally providing accurate quantitative information.

The authors have been working more than a quarter century to establish methods for the verified computation of solutions for partial differential equations, mainly for nonlinear elliptic problems of the form $-\Delta u=f(x,u,\nabla u)$ with Dirichlet boundary conditions. Here, by “verified computation” is meant a computer-assisted numerical approach for proving the existence of a solution in a close and explicit neighborhood of an approximate solution. The quantitative information provided by these techniques is also significant from the viewpoint of a posteriori error

estimates for approximate solutions of the concerned partial differential equations in a mathematically rigorous sense.

In this monograph, the authors give a detailed description of the verified computations and computer-assisted proofs for partial differential equations that they developed. In Part I, the methods mainly studied by the authors Nakao and Watanabe are presented. These methods are based on a finite dimensional projection and constructive a priori error estimates for finite element approximations of the Poisson equation. In Part II, the computer-assisted approaches via eigenvalue bounds developed by the author Plum are explained in detail. The main task of this method consists of establishing eigenvalue bounds for the linearization of the corresponding nonlinear problem at the computed approximate solution. Some brief remarks on other approaches are also given in Part III. Each method in Parts I and II is accompanied by appropriate numerical examples that confirm the actual usefulness of the authors' methods. Also in some examples practical computer algorithms are supplied so that readers can easily implement the verification programs by themselves.



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